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Statistics of the One-Electron Current in a One-Dimensional Mesoscopic Ring at Arbitrary Magnetic Fields

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The set of moments and the distribution function of the one-electron current in a one-dimensional disordered ring with arbitrary magnetic flux are calculated.

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1. INTRODUCTION

The theoretical work of ref. 1 together with recent experimental results[2,3] have stirred a strong interest in the problem of the persistent current in a mesoscopic metal ring immersed in a magnetic field. This current has been computed using various approaches.[4-11] Some works[4-7] considered the experimental situation of a ring of finite thickness such that the number of transverse channels is much greater than one. This allowed the authors to use the methods of weak-localization theory. In this case one has also to take into account the electron–electron interaction.[8]

On the other hand, the computation of the one-electron current in an idealized one-dimensional disordered ring is also of interest, at least from the theoretical point of view. One-dimensional localization effects lead in this case to a nontrivial current dependence on the magnetic flux (see ref. 9 and below). In ref. 10 such a calculation was performed, but only in the

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weak magnetic field case. In ref. 11 the one-electron current averaged over an ensemble of rings was derived nonperturbatively using Grassmann matrix integration. However, the unexplicit form of the resulting expression and the need for tedious computations do not allow one to check the validity of some approximations.

A new path integral approach to the study of one-dimensional localization was developed in ref. 9. Along with the multipoint density correlators, this new method allowed one to compute the averaged absolute values $\langle |I| \rangle$ of the one-electron current I in a disordered metal ring with arbitrary magnetic flux.

In the present paper we show that the method introduced in ref. 9 allows us to reconstruct completely the distribution function $\mathscr{P}_\Phi(I)$ over an ensemble of one-dimensional rings with given magnetic flux Φ. Simple explicit expressions for the moments $\langle I^{2n} \rangle$ of the current I are also obtained.

2. PATH INTEGRAL REPRESENTATION

Let us recall the main steps of the path-integral approach introduced in ref. 9. The Schrödinger equation

$$(\hat{H} - k^2)\,\psi = \left(-\frac{d^2}{dx^2} + U(x) - k^2 \right) \psi = 0 \tag{2.1}$$

maps the two-dimensional space of the initial conditions $(\psi'(x_0) + ik\psi(x_0),\ \psi'(x_0) - ik\psi(x_0))$ to the two-dimensional space of the solutions at the point x through the matrix

$$T(x, x_0) = e^{ik(x - x_0)\sigma^z}\, \mathscr{T}(x, x_0)\, e^{ik(x - x_0)\sigma^z} \tag{2.2}$$

where $\mathscr{T}(x, x_0)$ obeys the following first-order equation:

$$\frac{d}{dx} \mathscr{T} = (i\varphi(x)\, s^z + \zeta^+(x)\, s^- + \zeta^-(x)\, s^+)\, \mathscr{T} \tag{2.3}$$

with

$$\varphi(x) = -\frac{1}{k}\, U(x), \qquad \zeta^\pm(x) = \pm\frac{i}{2k}\, U(x)\, e^{\pm 2ikx} \tag{2.4}$$

Here $s^z = \sigma^z/2$ and $s^\pm = (\sigma^x \pm i\sigma^y)/2$ are the usual spin operators. It has been shown[12] (see also ref. 9 for greater detail) that in the limit

$$kl \gg 1 \tag{2.5}$$

where l is the mean free path, the fields $\varphi(x)$ and $\zeta^{\pm}(x)$ can be considered as statistically independent. If the initial potential $U(x)$ is a Gaussian random function of x with correlator

$$\langle U(x)\, U(x') \rangle = D\delta(x - x') \tag{2.6}$$

then the averaging weight over the fields $\varphi(x)$ and $\zeta^{\pm}(x)$ has the form

$$\mathscr{D}\varphi(x)\,\mathscr{D}\zeta^{\pm}(x) \exp\left\{ -l \int dx\, [\tfrac{1}{8}\varphi^2(x) + \zeta^+(x)\,\zeta^-(x)] \right\} \tag{2.7}$$

where $l = 4k^2/D$ is the localization length. It has been shown[9, 13-15] that the following change of variables in (2.7)

$$\begin{aligned} i\varphi &= i\rho + 2\psi^+\psi^- \\ \zeta^- &= \dot{\psi}^- - i\rho\psi^- - \psi^+(\psi^-)^2 \\ \zeta^+ &= \psi^+ \end{aligned} \tag{2.8}$$

brings the operator $\mathscr{T}(x, x_0)$ in the form of a product of usual matrix exponentials:

$$\begin{aligned} \mathscr{T}(x, x_0) = \exp[s^+\psi^-(x)] \exp\left[is^z \int_{x_0}^{x} dt\, \rho(t) \right] \\ \times \exp\left\{ s^- \int_{x_0}^{x} dt\, \psi^+(t) \exp\left[i \int_{x_0}^{t} dt'\, \rho(t') \right] \right\} \end{aligned} \tag{2.9}$$

This statement can be checked by deriving the evolution equation for the operator (2.9) and comparing it with (2.3). The field $\psi^-(x)$ is assumed to obey the initial condition $\psi^-(x_0) = 0$, thus providing the equality $\mathscr{T}(x_0, x_0) = 1$. Under a proper regularization, which is required by physical considerations (see ref. 9), the Jacobian of the transformation (2.8) is seen to be equal to

$$\mathscr{J} \propto \exp\left[-\frac{i}{2} \int_{-L}^{L} dt\, \rho(t) \right] \tag{2.10}$$

The surface of integration in the space of the complex fields φ, $\zeta^{\pm}$ is defined by the equations

$$\operatorname{Im} \varphi = 0, \qquad \zeta^- = (\zeta^+)^* \tag{2.11}$$

where the asterisk denotes complex conjugation. This surface can be deformed to the standard one

$$\operatorname{Im} \rho = 0, \qquad \psi^- = (\psi^+)^* \tag{2.12}$$

if all the quantities to be averaged are written in a form which allows analytical continuation from the surface (2.11) to the whole complex space of field configurations (for more details see refs. 9 and 13). This requirement turns out to be fairly constructive.

3. CALCULATION OF THE CURRENT MOMENTS AND OF THE CURRENT DISTRIBUTION FUNCTION

In an appropriate gauge the wave function of an electron moving in a metal ring of size $2L$ immersed in a magnetic flux Φ, measured in units of flux quanta, obeys Eq. (2.1). Topology and flux dependence are then encoded in the boundary conditions

$$(\psi'(L) \pm ik\psi(L)) = e^{2\pi i\Phi}(\psi'(-L) \pm ik\psi(-L)) \tag{3.1}$$

The mean value of a function $f(I)$ of the current I can be defined as follows:

$$\langle f(I) \rangle = \left\langle \frac{2\pi k}{L} \sum_n \delta(E - E_n)\, f(j_n) \right\rangle, \qquad \text{where} \quad j_n = -\frac{1}{2\pi} \frac{\partial E_n}{\partial \Phi} \tag{3.2}$$

Here $E = k^2$ is the electron energy and E_n are the eigenvalues of the Hamiltonian (2.1) with boundary conditions (3.1), which can be written in terms of the matrix $T \equiv T(L, -L)$:

$$\det(T - e^{2\pi i\Phi}) = 0 \tag{3.3}$$

The matrix $\mathcal{T} \equiv \mathcal{T}(L, -L)$ satisfies the "unitarity" conditions

$$\sigma^z \mathcal{T}^\dagger \sigma^z = \mathcal{T}^{-1}, \qquad \det \mathcal{T} = 1 \tag{3.4}$$

and therefore admits the following parametrization:

$$\mathcal{T} = \begin{pmatrix} e^{i\alpha_s} \cosh \Gamma & e^{i\beta_s} \sinh \Gamma \\ e^{-i\beta_s} \sinh \Gamma & e^{-i\alpha_s} \cosh \Gamma \end{pmatrix} \tag{3.5}$$

Here α_s, β_s, and Γ are by construction (see ref. 3) slowly varying real functions of L. Substituting the parametrization (3.5) into (2.2), we obtain from (3.3) the equation determining the set of eigenvalues E_n,[10]

$$\tau(E) \equiv \cosh \Gamma \cos(\alpha_s + kL) = \cos 2\pi\Phi \tag{3.6}$$

Let us start the computation of $\langle I^{2n} \rangle$:

$$\langle I^{2n} \rangle = \left\langle \frac{2\pi k}{L} \sum_n \delta(E - E_n) j_n^{2m} \right\rangle = \left\langle \frac{2\pi k}{L} \delta(\tau(E) - \cos 2\pi\Phi) \frac{\sin^{2m} 2\pi\Phi}{|\tau'(E)|^{2m-1}} \right\rangle \tag{3.7}$$

Here (3.3) and (3.6) have been taken into account. The δ-function can be eliminated in (3.7) using the following consideration: for $kL \gg 1$ the result of the average (3.7) does not change when L varies on a a scale much less than l. Then $\langle I^{2m} \rangle$ must coincide with its average over an interval ΔL of length L:

$$\langle I^{2m} \rangle_L = \frac{1}{\Delta L} \int_L^{L+\Delta L} dL \, \langle I^{2m} \rangle_L, \qquad \text{where} \quad \frac{1}{k} \ll \Delta L \ll l \tag{3.8}$$

We can interchange the order of the two averages; then, using the (approximate) constancy of the variables Γ, α_s, and β_s on the interval ΔL, we obtain[4]

$$\langle I^{2m} \rangle = (I_0 \sin 2\pi\Phi)^{2m} \left\langle \frac{1}{(\sinh^2 \Gamma + \sin^2 2\pi\Phi)^m} \right\rangle \tag{3.9}$$

where we have set $I_0 = 2k/L$. Equation (3.9) can be rewritten in the form

$$\langle I^{2m} \rangle = \frac{2}{(m-1)!} (I_0 \sin 2\pi\Phi)^{2m} \int_0^\infty d\mu \, \mu^{2m-1} \langle e^{-\mu^2(\sinh^2\Gamma + \sin^2 2\pi\Phi)} \rangle \tag{3.10}$$

It is important to notice that $\sinh^2 \Gamma$ can be expressed in terms of the elements of the matrix $\mathcal{T}$ without using any complex conjugation:

$$\sinh^2 \Gamma = (1 \quad 0) \, \mathcal{T}^t s^- \mathcal{T} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{3.11}$$

where t denotes the usual matrix transposition. Thus the above-mentioned analytic continuation from the surface (2.11) is possible. Substituting (2.9) into (3.10), we obtain

$$\sinh^2 \Gamma = \psi^-(L) \int_{-L}^{L} dt \, \psi^+(t) \exp\left[-i \int_t^L dt' \, \rho(t') \right] \tag{3.12}$$

The average (3.10) is performed using the weight

$$\mathcal{D}\rho \, \mathcal{D}\psi^\pm \, e^{-S'(\rho, \psi^\pm)} \tag{3.13}$$

[4] Such a procedure, which was proposed in ref. 9, seems to be equivalent to the rings-ensemble averaging of ref. 11. See also ref. 16.

where the action $S'(\rho, \psi^{\pm})$ is obtained from (2.7) after the substitution (2.8), taking into account the Jacobian (2.10), and reads

$$S'(\rho, \psi^{\pm}) = l \int_{-L}^{L} dx \left[\frac{1}{8} \rho^2 + \psi^+ \dot{\psi}^- - \frac{3}{2} i\rho\psi^+\psi^- - \frac{3}{2} (\psi^+\psi^-)^2 \right] + \frac{i}{2} \int_{-L}^{L} dx\, \rho \tag{3.14}$$

This action has the form of a $(0+1)$-dimensional Schwinger model and the $\mathscr{D}\psi^{\pm}$ integration in (3.10) can be performed using the so-called "bosonization" method,[17] representing

$$\exp\left[\tfrac{3}{2} l \int_{-L}^{L} dx\, (\psi^+\psi^-)^2 \right] = \int \mathscr{D}\eta \exp\left[-\tfrac{3}{2} l \int_{-L}^{L} dx\, (\eta^2 + 2\eta\psi^+\psi^-) \right] \tag{3.15}$$

and

$$\exp(-\mu^2 \sinh^2 \Gamma) = \frac{1}{\pi} \int dz\, dz^* \exp(-|z|^2) \times \exp\left\{ -i\mu z \psi^-(L) - i\mu z^* \int_{-L}^{L} dx\, \psi^+(x) \times \exp\left[-i \int_{x}^{L} dt\, \rho(t) \right] \right\} \tag{3.16}$$

We eliminate the $\rho\psi^+\psi^-$ and $\eta\psi^+\psi^-$ interaction terms through the following gauge transformation:

$$\psi^{\pm}(x) \rightarrow \psi^{\pm}(x) \exp\left[\pm\tfrac{3}{2} \int_{-L}^{x} dt\, (2\eta - i\rho) \right] \tag{3.17}$$

which has the Jacobian

$$\mathscr{J}_R \propto \exp\left[-\tfrac{3}{4} \int_{-L}^{L} dt\, (2\eta - i\rho) \right] \tag{3.18}$$

The $\mathscr{D}\psi^{\pm}$ integration becomes Gaussian and can be easily performed. Introducing the variable $\xi(x)$ and denoting the x derivative with a dot, we have

$$\dot{\xi} = -3\eta + \frac{i}{2}\rho, \qquad \xi(L) = 0, \qquad \mathscr{D}\rho\, \mathscr{D}\eta \propto \mathscr{D}\rho\, \mathscr{D}\xi \tag{3.19}$$

and performing the Gaussian $\mathscr{D}\rho$ integration, we come to the following expression for $\langle I^{2m}\rangle$:

$$\langle I^{2m}\rangle = \frac{2}{\pi(m-1)!}(I_0 \sin 2\pi\Phi)^{2m} \int_0^\infty d\mu\, \mu^{2m-1}$$
$$\times \int dz\, dz^*\, \exp(-\mu^2 \sin^2 2\pi\Phi - |z|^2)\, \mathscr{N} \exp\left(-\frac{L}{2l}\right) \int_{\xi(L)=0} \mathscr{D}\xi$$
$$\times \exp\left\{-\frac{l}{4}\int_{-L}^{L} dx \left[\dot{\xi}^2 + \frac{4}{l^2}\mu^2 |z|^2 \exp(-\xi)\right]\right\} \exp\left[-\frac{\xi(-L)}{2}\right]$$
$$= \frac{l^{2m}}{2^{2m-1}(m-1)!}(I_0 \sin 2\pi\Phi)^{2m} \left[\exp\left(-\frac{L}{2l}\right)\right] \int_0^\infty \frac{dr}{r^{2m-1}}$$
$$\times [\exp(-r^2)]\, \langle \Upsilon_2^{(m)}(\xi, r)|\, \exp(-2L\hat{H})\, |\Upsilon_1(\xi)\rangle \qquad (3.20)$$

in terms of usual quantum mechanical matrix elements with Hamiltonian

$$\hat{H} = -\frac{1}{l}\frac{d^2}{d\xi^2} - \frac{l}{4}e^{-\xi} \qquad (3.21)$$

where the ket and bra wave functions are

$$\Upsilon(\xi) = e^{-\xi/2}, \qquad \Upsilon_2^{(m)}(\xi, r) = \exp\left[-\xi\left(m - \frac{1}{2}\right)\right] \exp\left(-\frac{(l^2 \sin^2 2\pi\Phi)e^{-\xi}}{4r^2}\right) \qquad (3.22)$$

The factor $\mathscr{N}$ in (3.20) is the normalization of the standard Feynman–Kac path integral; together with $e^{-L/2l}$ it provides the equality $\langle 1\rangle = 1$. Using the complete set of eigenfunctions of $\hat{H}$

$$f_\nu(\xi) = \frac{2}{\pi}(\nu \sinh 2\pi\nu)^{1/2} K_{2i\nu}(le^{-\xi/2})$$
$$\hat{H}f_\nu(\xi) = -\frac{1}{l}\nu^2 f_\nu(\xi), \qquad \langle f_\nu | f_{\nu'}\rangle = \delta(\nu - \nu') \qquad (3.23)$$

where K_μ is the standard notation for the modified Bessel function, we obtain, after some arithmetic,

$$\langle I^{2m}\rangle = (I_0 \sin 2\pi\Phi)^{2m} \frac{(-1)^{m-1}}{(m-1)!}\left(\frac{\partial}{\partial(\sin^2 2\pi\Phi)}\right)^{m-1} \langle I^2 (I_0 \sin 2\pi\Phi)^{-2}\rangle \qquad (3.24)$$

and

$$\langle I^2 \rangle = \frac{2e^{-L/2l}}{\sqrt{\pi}\,(2L/l)^{3/2}}\,(I_0 \sin 2\pi\Phi)^2 \int_0^{\infty} dx \,\frac{x e^{-(l/2L)\,x^2}}{(\sinh^2 x + \sin^2 2\pi\Phi)^{1/2}}$$
$$\times \log\left[1 + 2\,\frac{\sinh^2 x}{\sin^2 2\pi\Phi} + 2\left(\frac{\sinh^2 x}{\sin^2 2\pi\Phi} + \frac{\sinh^4 x}{\sin^4 2\pi\Phi}\right)^{1/2}\right] \quad (3.25)$$

In order to reconstruct the distribution function $\mathscr{P}_{\Phi}(I)$, we use the following identity:

$$\mathscr{P}_{\Phi}(J) = \frac{1}{\pi} \lim_{\varepsilon \to 0^+} \operatorname{Im} \int \frac{\mathscr{P}_{\Phi}(I)\, dI}{J - I + i\varepsilon} = \frac{1}{\pi J} \lim_{\varepsilon \to 0^+} \operatorname{Im} \sum_{n=0}^{\infty} \frac{\langle I^{2m} \rangle}{(J + i\varepsilon)^{2n}} \quad (3.26)$$

It is convenient to compute the sum in (3.26) considering it as the result of an analytical continuation in J from the imaginary positive semiaxis. Substituting (3.24) into (3.26), we see that for such J the summation gives a well-defined translation operator in the variable $\sin^2 2\pi\Phi$ acting on $\langle I^2 (I_0 \sin 2\pi\Phi)^{-2} \rangle$. Performing the analytical continuation to the real axis, we obtain

$$\mathscr{P}_{\Phi}(I) = 0 \qquad \text{for} \quad |I| > I_0 \quad (3.27)$$

and

$$\mathscr{P}_{\Phi}(I) = \frac{2e^{-L/2l}}{\sqrt{\pi}\,(2L/l)^{3/2}}\,\frac{(I_0 \sin 2\pi\Phi)^2}{|I|^3}$$
$$\times \int_{\lambda(I)}^{+\infty} dx\, \frac{x e^{-(l/2L)\,x^2}}{[\sinh^2 x - \sinh^2 \lambda(I)]^{1/2}} \qquad \text{for} \quad |I| < I_0 \quad (3.28)$$

where

$$I_0 \equiv \frac{2k}{L} \qquad \text{and} \qquad \lambda(I) = \sinh^{-1}\left[|\sin 2\pi\Phi| \left(\frac{I_0^2}{I^2} - 1\right)^{1/2}\right] \quad (3.29)$$

It can be checked that this distribution function reproduces all the moments $\langle I^{2m} \rangle$ as well as the result for $\langle |I| \rangle$ obtained in ref. 9.

In the limit $I \to 0$ we get

$$\lambda \sim \log\left(2\,\frac{|I_0 \sin 2\pi\Phi|}{|I|}\right) \quad (3.30)$$

and

$$\mathscr{P}_\Phi(I) \sim \frac{\lambda \exp[-(l/2L)(\lambda - L/l)^2]}{|I|\,(2L/l)^{3/2}} \frac{\Gamma(\lambda/(2L/l)+1/2)}{\Gamma(\lambda/(2L/l)+1)} \tag{3.31}$$

in the proximity of the maximum $\lambda \sim L/l$. In the limit $L/l \to +\infty$ we get for the quantity λ the normal distribution

$$\mathscr{P}_\Phi(I)\,dI = \frac{\exp[-(l/2L)(\lambda - L/l)^2]}{(2\pi L/l)^{1/2}}\,d\lambda \tag{3.32}$$

Thus in the thermodynamic limit the fluctuations of λ are suppressed and λ becomes a nonrandom quantity. This fact is deeply connected with an earlier result[18] about the asymptotically normal distribution of the logarithm of the static resistivity (see also ref. 12). In both cases we are dealing with the response of the system to an external field. In our case $\sin^2 2\pi\Phi$ can formally assume an arbitrary value and in some sense we are considering a nonlinear response. However, we see from (3.32) that in the limit $L/l \to +\infty$ the response becomes effectively linear.

When $I \to I_0$, (3.28) gives

$$\mathscr{P}_\Phi(I_0) = \frac{2e^{-L/2l}}{\sqrt{\pi}(2L/l)^{3/2}} \frac{\sin^2 2\pi\Phi}{I_0} \int_0^\infty dx\, \frac{xe^{-(l/2L)x^2}}{\sinh x}$$

It is worth noting that in the limit $2\pi\Phi \to 0$, when $l/L < \infty$ is fixed, all the moments of the current I tend to zero. This seems natural since in a given potential without symmetries ($l/L < \infty$) and with zero magnetic field all the stationary states of the electron in the ring can be described by real wave functions. The corresponding quantum mechanical expectation values of the current operator are equal to zero. On the other hand, if we take simultaneously the limit $l/L \to \infty$ (free motion case), we can obtain a nonzero result.

Let us also note that the formal substitution $I_0 = 1$, $\sin^2 2\pi\Phi = 1$, and $I^2 \to T$ in (3.28) gives us the distribution function for the transmission coefficient T. In the limit $L/l \to \infty$ this reproduces the known results[19, 20] for the moments $\langle T^n \rangle$, but our formula is valid for finite values of L/l as well (the only limitation is that the sample length $2L$ and the localization length l be great in comparison to the wavelength $1/k$).

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